

# Overview of the Tutorial

## 1. Introduction

- " Transposing programming concepts to the analysis of living processes

## 2. Rule-based modeling of biochemical systems

- " Syntax: molecules, reactions, regulations, SBML/SBGN Biocham notations
- " Semantics: Boolean, Differential and Stochastic interpretations of reactions
- " *Static analyses*: consistency, influence graph circuits, protein functions,&
- " Examples in cell signaling, gene expression, virus infection, cell cycle

## 3. Temporal Logic based formalization of biological properties

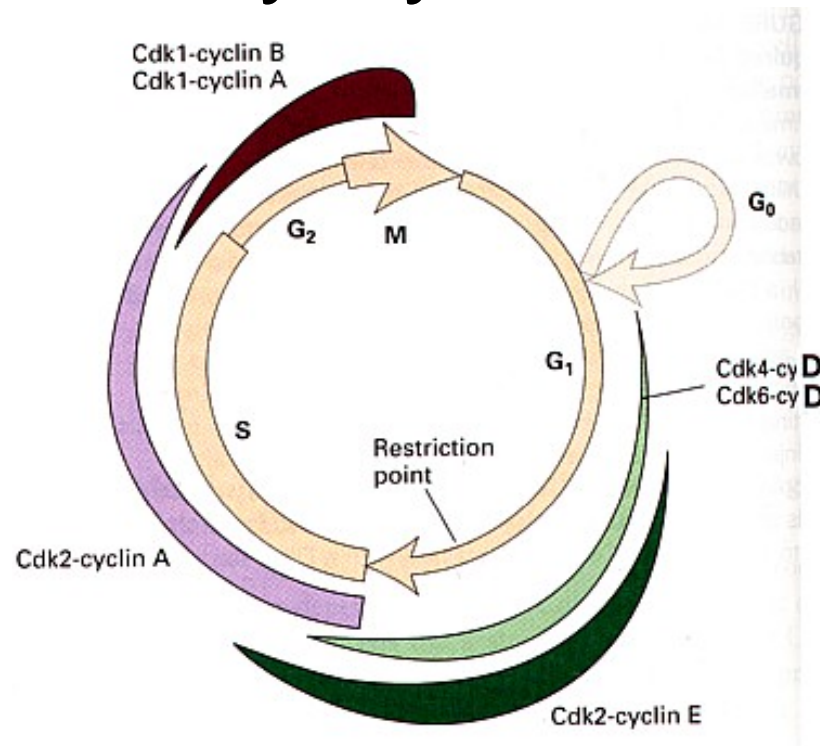
- " *Qualitative model-checking* in propositional Computation Tree Logic CTL
- " Quantitative model-checking in Linear Time Logic LTL(R)
- " *Parameter search* in high dimension w.r.t. LTL(R) specifications
- " *Robustness and sensitivity analyses* w.r.t. LTL(R) specifications

## 4. Conclusion

# Cell Cycle Control by Cyclins: $G_1 \rightarrow S \rightarrow G_2 \rightarrow M$



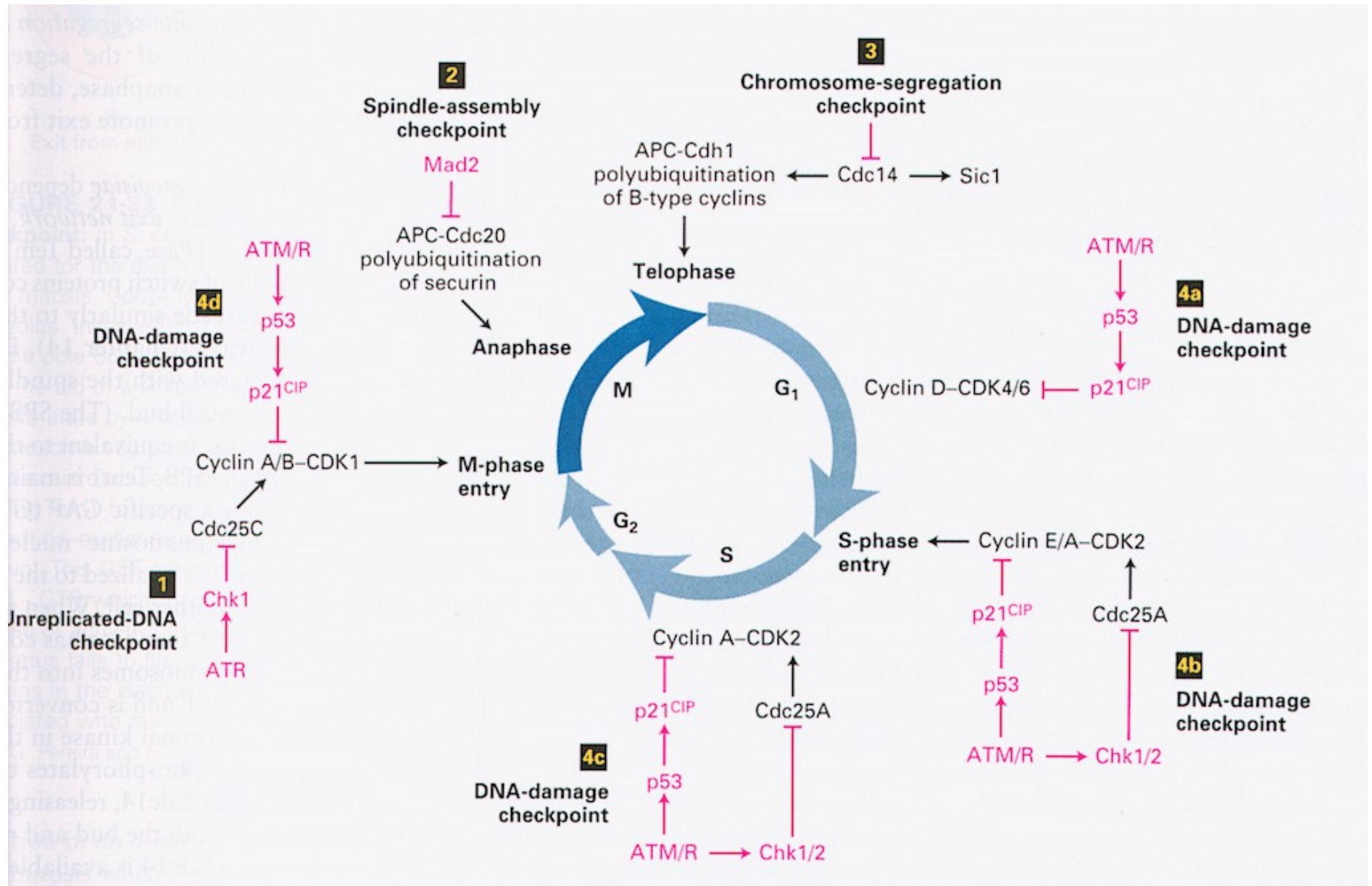
Sir Paul Nurse  
Nobel prize 2001



**G<sub>1</sub>: Cdk4-CycD**  
**Cdk6-CycD**  
**Cdk2-CycE**

**S: Cdk2-CycA**

**G<sub>2</sub>,M: Cdk1-CycA**  
**Cdk1-CycB (MPF)**



[illegible]

# Kohn's map detail for Cdk2

Complexation with CycA and CycE

Biocham Rule Patterns:

$$\text{cdk2} \sim \$P + \text{cycA} - \$C \Rightarrow \text{cdk2} \sim \$P - \text{cycA} - \$C$$

where  $\$C$  in  $\{\_, \text{cks1}\}$ .

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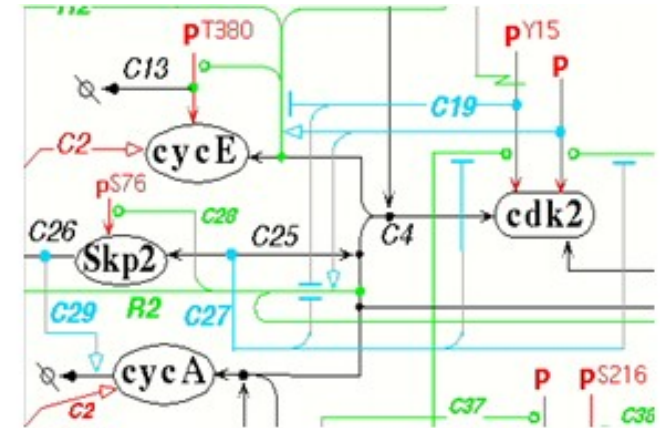
$$\text{p57} + \text{cdk2} \sim \$P - \text{cycA} - \$C \Rightarrow \text{p57} - \text{cdk2} \sim \$P - \text{cycA} - \$C$$

where  $\$C$  in  $\{\_, \text{cks1}\}$ .

$$\text{cycE} - \$C = [\text{cdk2} \sim \{p2\} - \text{cycE} - \$S] \Rightarrow \text{cycE} \sim \{T380\} - \$C$$

where  $\$S$  in  $\{\_, \text{cks1}\}$  and  $\$C$  in  $\{\_, \text{cdk2} \sim ?, \text{cdk2} \sim ? - \text{cks1}\}$

Total: 147 rule patterns 2733 expanded rules [Chiaverini Danos 03]



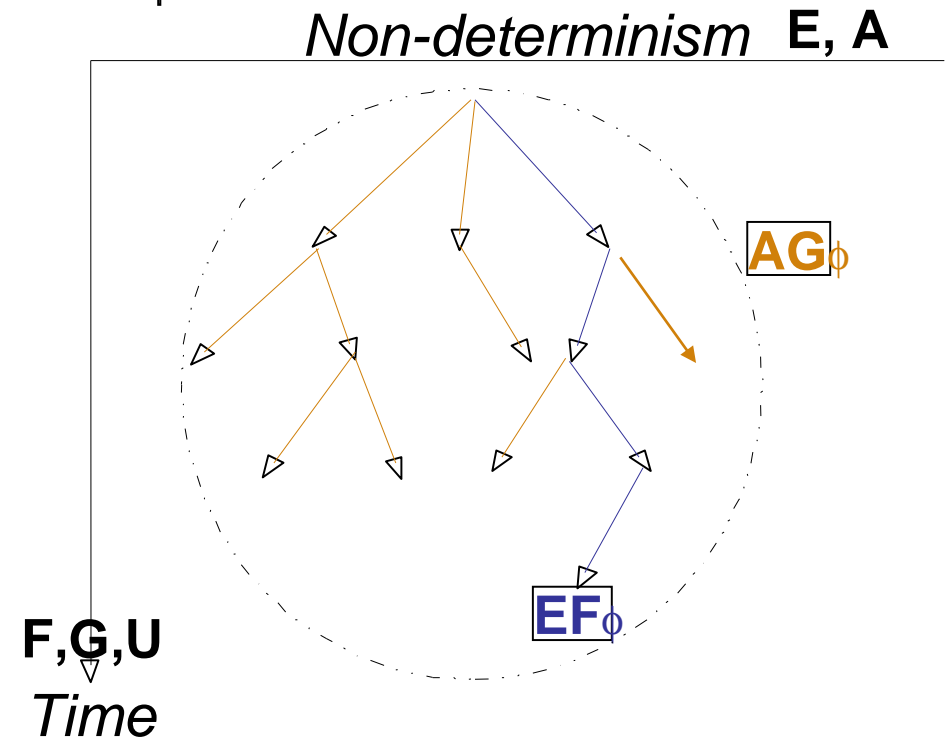


# Computation Tree Logic CTL

Temporal logics extend classical logic with **modal operators** for time & non-det.

Introduced for program verification by [Pnueli 77]

Non-det. Time	<b>E</b> exists	<b>A</b> always
<b>X</b> next time	<b>EX</b> ( $\varphi$ )	<b>AX</b> ( $\varphi$ )
<b>F</b> finally	<b>EF</b> ( $\varphi$ ) $\neg \mathbf{AG}(\neg \varphi)$	<b>AF</b> ( $\varphi$ ) <i>liveness</i>
<b>G</b> globally	<b>EG</b> ( $\varphi$ ) $\neg \mathbf{AF}(\neg \varphi)$	<b>AG</b> () <i>safety</i>
<b>U</b> until	<b>E</b> ( $\varphi_1 \mathbf{U} \varphi_2$ )	<b>A</b> ( $\varphi_1 \mathbf{U} \varphi_2$ )



# Biological Properties formalized in CTL

## (1/3)

About reachability:

- " Can the cell produce some protein P? **reachable**(P) == **EF**(P)
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About pathways:

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- " Is state  $s_2$  a necessary checkpoint for reaching state  $s$ ?

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- " Is  $s_2$  always a checkpoint for  $s$ ? **AG**(¬ $s$  → **checkpoint**( $s_2, s$ ))

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needs to combine CTL with search [Chan 00, Calzone-Chabrier-Fages-Soliman 05,  
Fages-Rizk 07 09].

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About oscillations:

- " Can the system exhibit a cyclic behavior w.r.t. the presence of P ?

**oscil(P) == EG ( (F  $\neg$ P) ^ (F P) )**

temporal operators not preceded by a path operator: CTL\* formula

approximation in CTL: **oscil(P) == EG ( (EF  $\neg$ P) ^ (EF P) )**

# Basic CTL Model-Checking Algorithm

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Model-checking algorithm in  $O(|K|^*|\phi|)$ .

**Complexity:** CTL model-checking is Ptime-complete,

# Symbolic CTL Model-Checking Algorithm

Represent *finite* Kripke structures using **Boolean constraints** for

- " **sets of states** as a boolean constraint  $c(V)$
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**Ordered Binary Decision Diagrams** OBDD [Bryant 85] provide *canonical forms* for Boolean formulas (decides SAT in NP, and equivalence TAUT in co-NP)

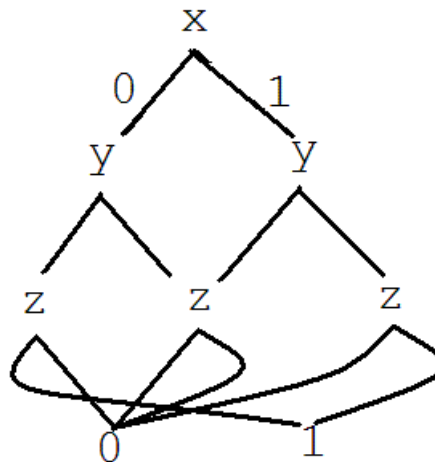
$(x \vee \neg y) \wedge (y \vee \neg z) \wedge (z \vee \neg x)$

and

$(x \vee \neg z) \wedge (z \vee \neg y) \wedge (y \vee \neg x)$

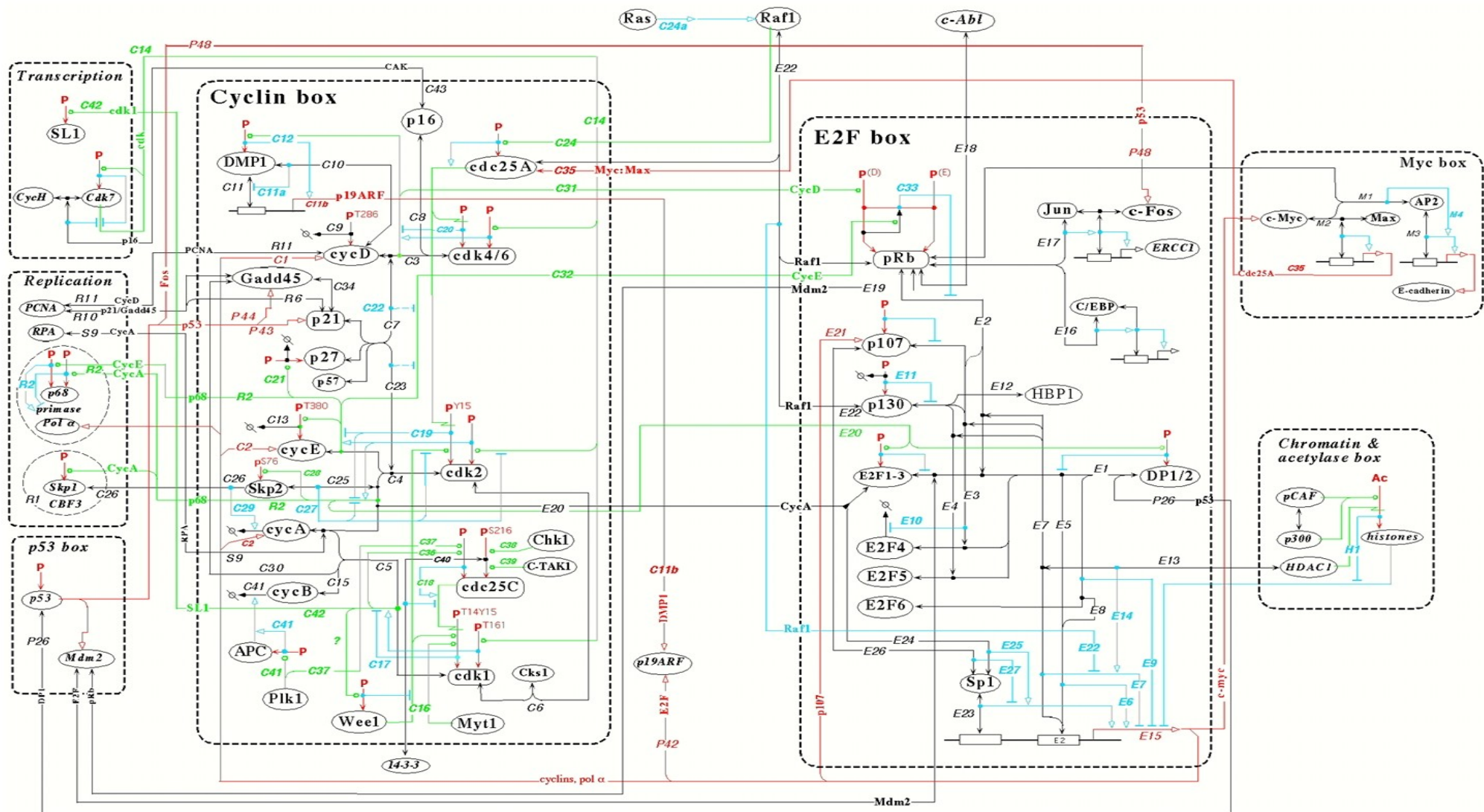
are equivalent, they

have the same BDD( $x, y, z$ )





# Mammalian Cell Cycle Control Map [Kohn 99]



# Mammalian Cell Cycle Control Benchmark

147-2733 rules, 165 proteins and genes, 500 variables,  $2^{500}$  states.

BIOCHAM NuSMV model-checker time in seconds: [Chabrier Fages 03 CMSB]

Initial state G2	Query:	Time:
	compiling	29s
Reachability G1	<b>EF CycE</b>	2s
Reachability G1	<b>EF CycD</b>	1.9s
Reachability G1	<b>EF PCNA-CycD</b>	1.7s
Checkpoint for mitosis complex	<b><math>\neg</math> EF (<math>\neg</math> Cdc25~{Nterm} U Cdk1~{Thr161}-CycB)</b>	2.2s
Oscillations CycA	<b>EG ( (EF <math>\rightarrow</math> CycA) &amp; (EF CycA))</b>	31.8s
Oscillations CycB	<b>EG ( (EF <math>\rightarrow</math> CycB) &amp; (EF CycB)) false !</b>	6s

# Linear Time Logic with Constraints LTL(R)

**Constraints** over *concentrations* and *derivatives* as formulae over the reals:

$$[M] > 0.2$$

$$[M] + [P] > [Q]$$

$$d([M])/dt < 0$$

**LTL(R)** formulae

minimum threshold value reached: **F**( $[M] > 0.2$ )

minimum threshold value reached and maintained: **FG**( $[M] > 0.2$ )

local maximum  $V$ : **F** ( $[M] < V$  & **F** ( $[M] = V$  & **F** ( $[M] < V$ ))

**F** ( $[M] > 2$  & **F** ( $d([M])/dt < 0$  & **F** ( $[M] < 2$  &  $d([M])/dt > 0$  & **F**( $d([M])/dt < 0$ ))))

oscil( $M, n$ ) defined as at least  $n$  alternances of the sign of the derivative

# LTL(R) Constraints with Real-time Variable

## LTL(R) formulae with real-time variable

Threshold value with a minimum delay

$$\mathbf{F}([M] > 0.2) \ \& \ \mathbf{G}(\text{Time} < 5 \Rightarrow [M] < 0.2)$$

Numerical data time series (for curve fitting)

$$\mathbf{F}(\text{Time} = 1 \ \& \ [M] = 0.05 \ \& \ \mathbf{F}(\text{Time} = 2 \ \& \ [M] = 0.12 \ \& \ \mathbf{F}(\text{Time} = 3 \ \& \ [M] = 0.25)))$$

Period constraint

$$\begin{aligned} \text{Period}(A, 75) = \exists t \exists v \ \mathbf{F}(\text{Time} = t \ \& \ [A] = v \ \& \ d([A])/dt > 0 \ \& \ \mathbf{X}(d([A])/dt < 0) \\ \ \& \ \mathbf{F}(\text{Time} = t + 75 \ \& \ [A] = v \ \& \ d([A])/dt > 0 \ \& \ \mathbf{X}(d([A])/dt < 0))) \end{aligned}$$

# Numerical Integration of ODE Models

$$dX/dt = f(X).$$

Initial conditions  $X_0$

Idea: discretize time  $t_0, t_1=t_0+\Delta t_0, t_2=t_1+\Delta t_1$ , &  
and compute a numerical trace  $(t_0, X_0, dX_0/dt), (t_1, X_1, dX_1/dt)$ , & ,  $(t_n, X_n, dX_n/dt)$

*Euler's method*  $t_{i+1}=t_i+\Delta t$   $X_{i+1}=X_i+f(X_i)*\Delta t$

error estimation  $E(X_{i+1})=|f(X_i)-f(X_{i+1})|*\Delta t$

*Runge-Kutta's method* intermediate computations at  $\Delta t/2$

adaptive step method:  $\Delta t_{i+1} = \Delta t_i/2$  while  $E > E_{max}$ , otherwise  $\Delta t_{i+1} = 2*\Delta t_i$

*Rosenbrock's implicit method* for stiff systems:

solve  $X_{i+1}=X_i+f(X_{i+1})*\Delta t$  by formal differentiation

# LTL(R) Trace-based Model Checking Algo

**Input:** An ODE system  $M$  given with initial conditions, an LTL(R) formula  $\phi$

*Hypothesis:* the formula can be checked over a **finite** period of time  $[0, T]$

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Add  $\phi_1 \mathbf{W} \phi_2$  to the states labelled by  $\phi_1 \wedge \phi_2$ , to the last state if it is labelled by  $\phi_1$ , and to the predecessors of states labelled by  $\phi_1 \mathbf{W} \phi_2$  while they satisfy  $\phi_1$ ,

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- " Return true if the **initial state is labelled by  $\phi$** , and false otherwise

# Naïve Parameter Search Algorithm

**Input:** an ODE model  $M(p)$  with  $n$  parameters  $p$  in range  $[pmin, pmax]$ ,  
an LTL(R) specification  $\phi$

**Output:** parameter values  $v$  such that  $M(v) \models \phi$   
or fail if no such values

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- " Test whether  $M(v) \models \phi$  by trace-based model checking



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or fail if no such values

- " Scan the parameter value space  $[pmin, pmax]^n$  with a fixed step
- " Test whether  $M(v) \models \phi$  by trace-based model checking
- " Return the first value set  $v$  which satisfies  $\phi$

# Naïve Parameter Search Algorithm

**Input:** an ODE model  $M(p)$  with  $n$  parameters  $p$  in range  $[pmin, pmax]$ ,  
an LTL(R) specification  $\phi$

**Output:** parameter values  $v$  such that  $M(v) \models \phi$   
or fail if no such values

- " Scan the parameter value space  $[pmin, pmax]^n$  with a fixed step
- " Test whether  $M(v) \models \phi$  by trace-based model checking
- " Return the first value set  $v$  which satisfies  $\phi$

**Exponential complexity** in  $O(s^n)$  where  $s$  is the maximum number of tried values in the range of  $n$  parameters

**Gradient-based methods** need a satisfaction degree for LTL(R) formulae&

# Cell Cycle Control Model [Tyson 91]

k1 for  $\_ \Rightarrow \text{Cyclin}$ .  
k2\*[Cyclin] for  $\text{Cyclin} \Rightarrow \_$ .  
k3\*[Cyclin]\*[Cdc2~{p1}] for  $\text{Cyclin} + \text{Cdc2~{p1}} \Rightarrow \text{Cdc2~{p1}} - \text{Cyclin~{p1}}$ .  
k4p\*[Cdc2~{p1} - Cyclin~{p1}] for  $\text{Cdc2~{p1}} - \text{Cyclin~{p1}} \Rightarrow \text{Cdc2} - \text{Cyclin~{p1}}$ .  
k4\*[Cdc2-Cyclin~{p1}]^2\*[Cdc2~{p1} - Cyclin~{p1}] for  
    Cdc2~{p1} - Cyclin~{p1} = [Cdc2-Cyclin~{p1}]  $\Rightarrow$  Cdc2-Cyclin~{p1}.  
k5\*[Cdc2-Cyclin~{p1}] for  $\text{Cdc2} - \text{Cyclin~{p1}} \Rightarrow \text{Cdc2~{p1}} - \text{Cyclin~{p1}}$ .  
k6\*[Cdc2-Cyclin~{p1}] for  $\text{Cdc2} - \text{Cyclin~{p1}} \Rightarrow \text{Cdc2} + \text{Cyclin~{p1}}$ .  
k7\*[Cyclin~{p1}] for  $\text{Cyclin~{p1}} \Rightarrow \_$ .  
k8\*[Cdc2] for  $\text{Cdc2} \Rightarrow \text{Cdc2~{p1}}$ .  
k9\*[Cdc2~{p1}] for  $\text{Cdc2~{p1}} \Rightarrow \text{Cdc2}$ .  
parameter(k1,0.015). parameter(k2,0.015). parameter(k3,200).  
parameter(k4p,0.018). parameter(k4,180). parameter(k5,0).  
parameter(k6,1). parameter(k7,0.6). parameter(k8,100).parameter(k9,100).  
present(Cdc2,1).

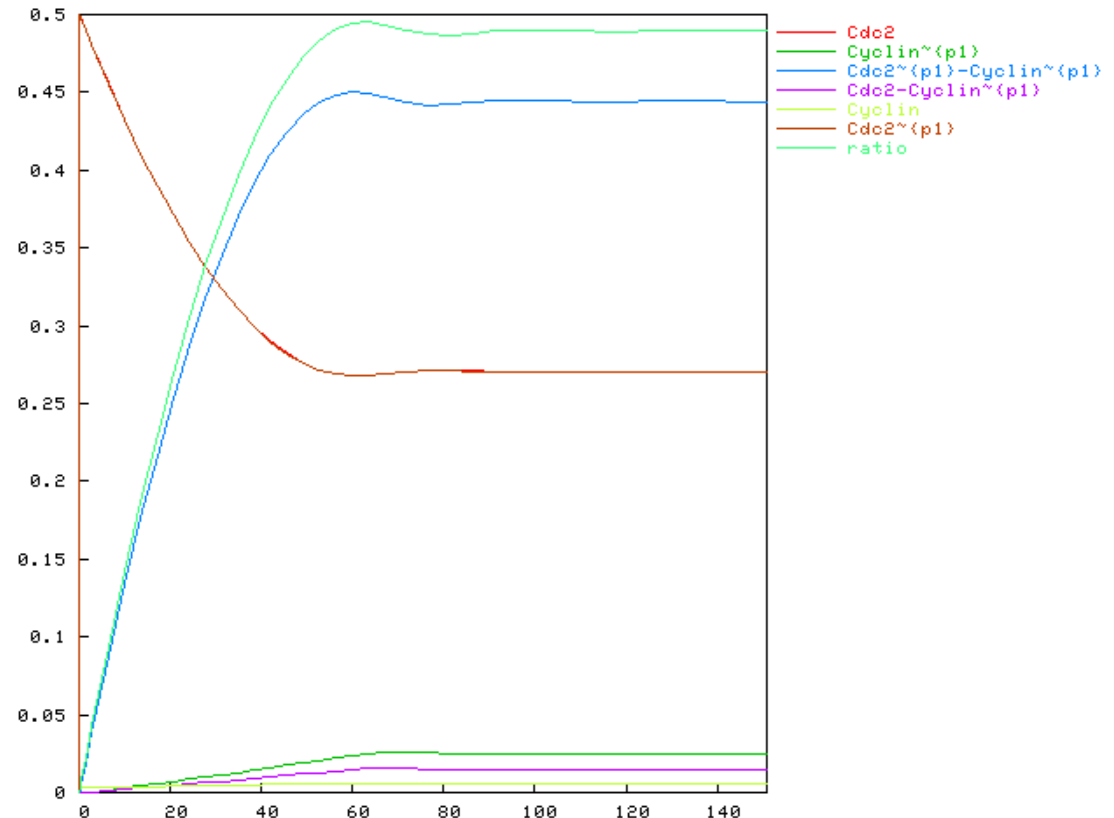
# Learning Parameters from Temporal Properties

```
biocham: learn_parameter([k3,k4],[(0,200),(0,200)],20,  
                        oscil(Cdc2-Cyclin~{p1},3),150).
```

# Learning Parameters from Temporal Properties

```
biocham: learn_parameter([k3,k4],[(0,200),(0,200)],20,  
    oscil(Cdc2-Cyclin~{p1}.3).150).
```

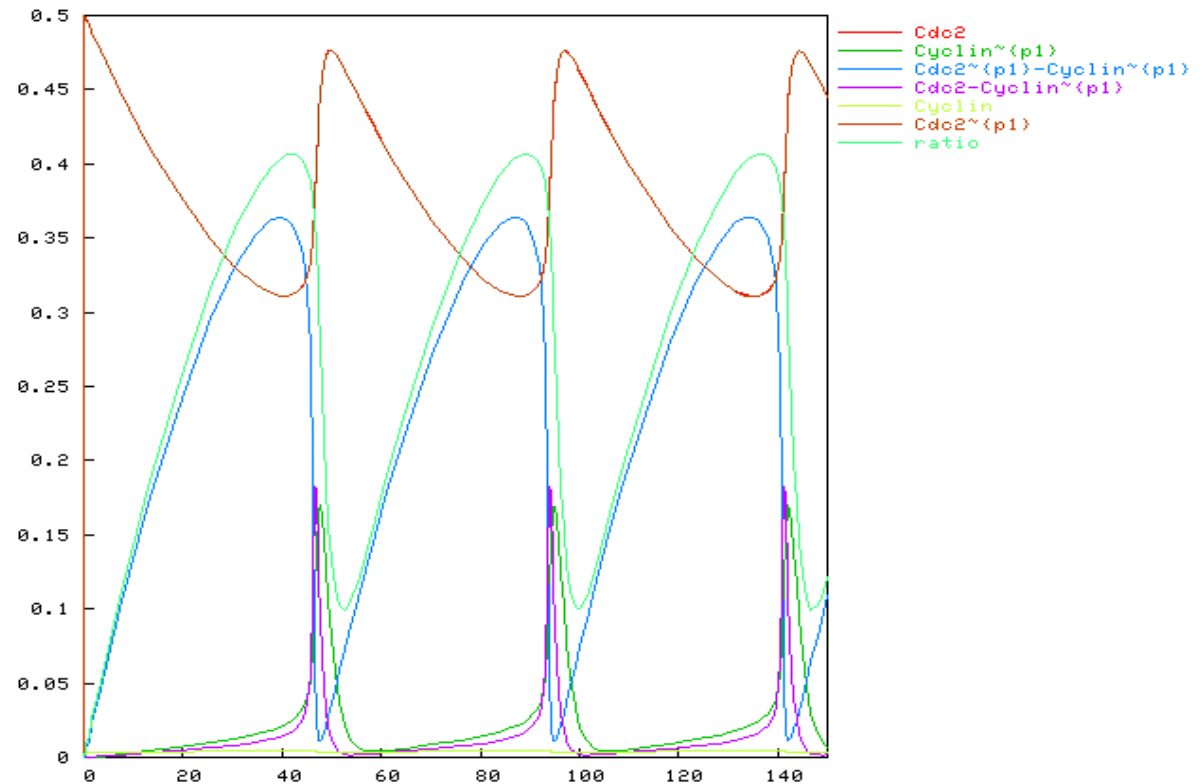
First values found :  
parameter(k3,10).  
parameter(k4,70).



# Learning Parameters from Temporal Properties

biocham: learn\_parameter([k3,k4],[(0,200),(0,200)],20,  
oscil(Cdc2-Cyclin~{p1},3) & F([Cdc2-Cyclin~{p1}]>0.15), 150).

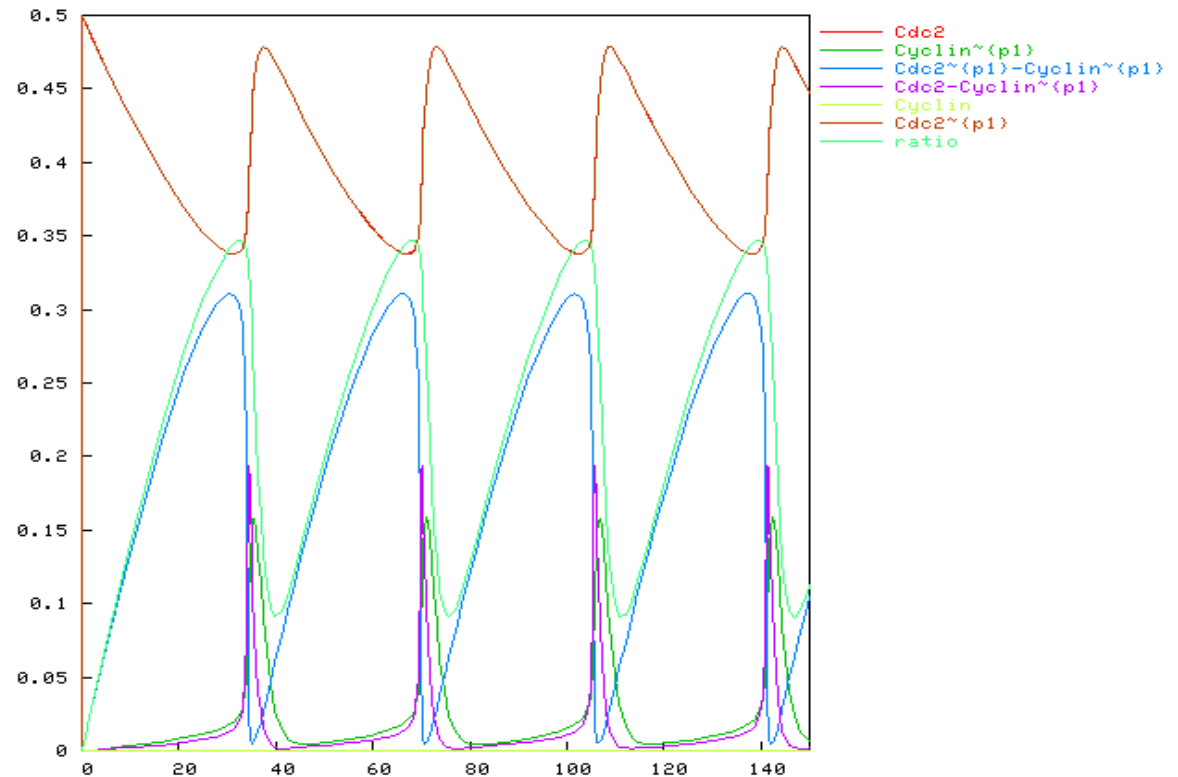
First values found :  
parameter(k3,10).  
parameter(k4,120).



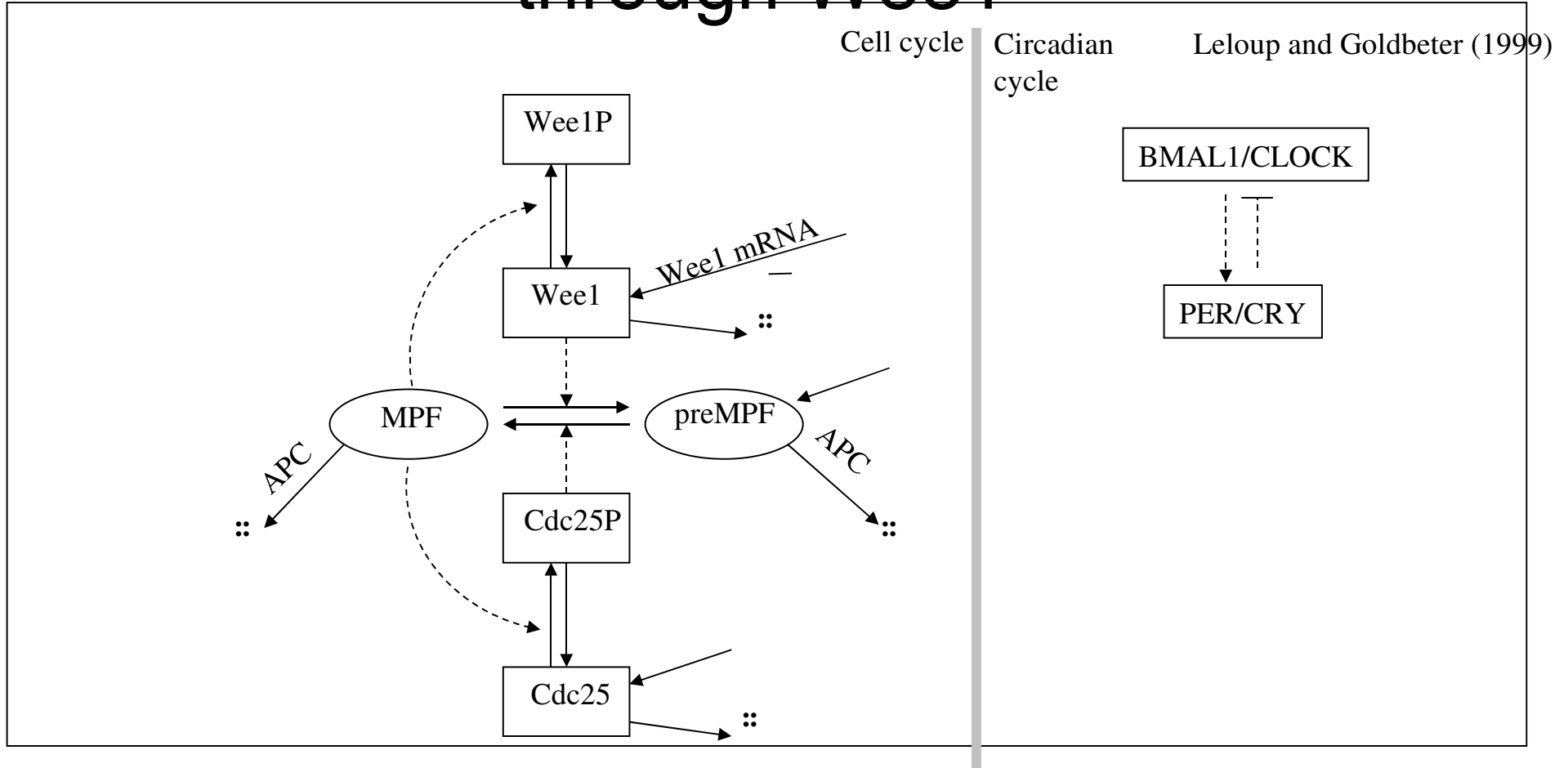
# Learning Parameters from Temporal Properties

```
biocham: learn_parameter([k3,k4],[(0,200),(0,200)],20,  
period(Cdc2-Cyclin~{p1},35), 150).
```

First values found:  
parameter(k3,10).  
parameter(k4,280).

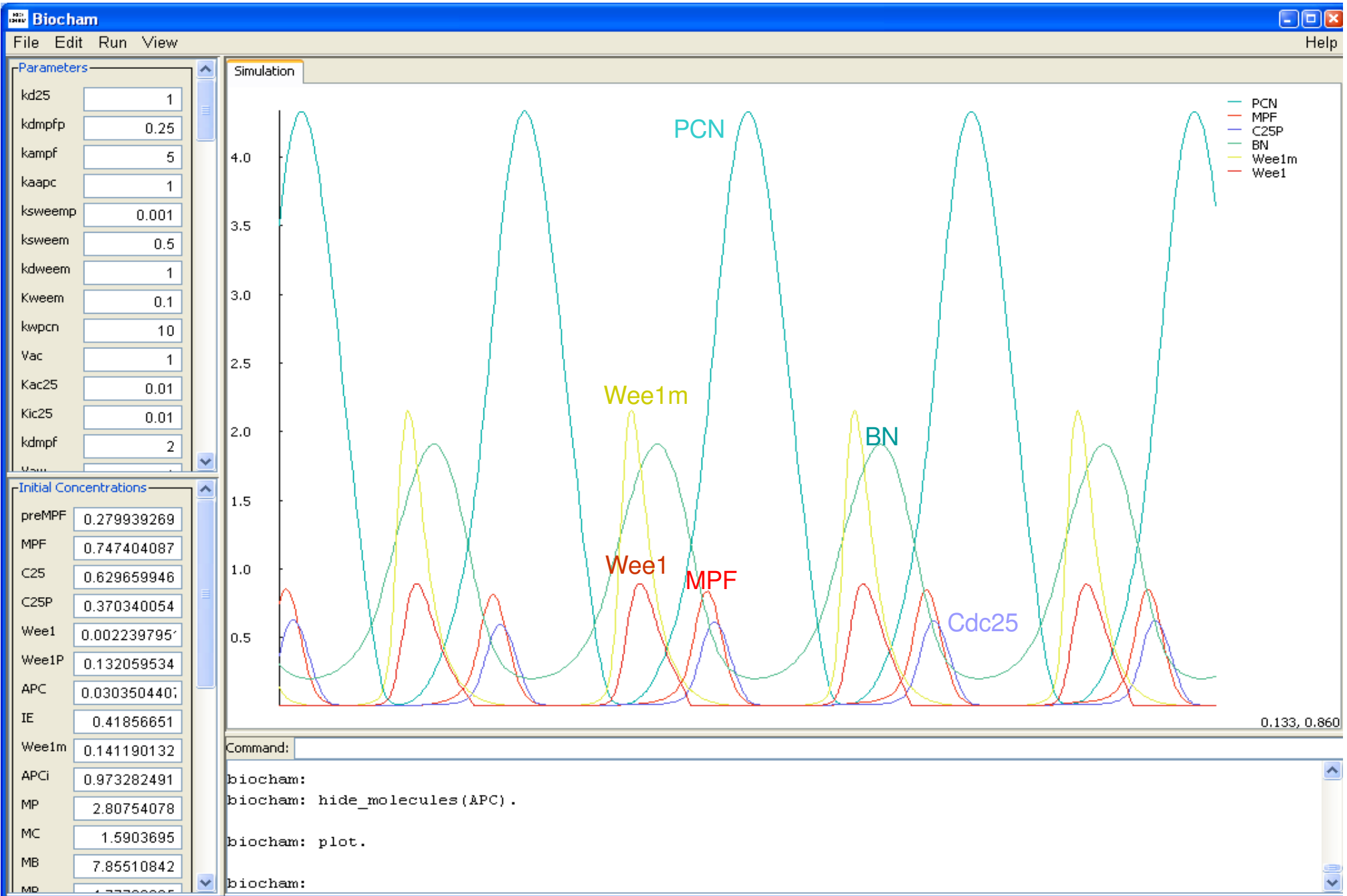


# Coupling Cell and Circadian Cycles through Wee1

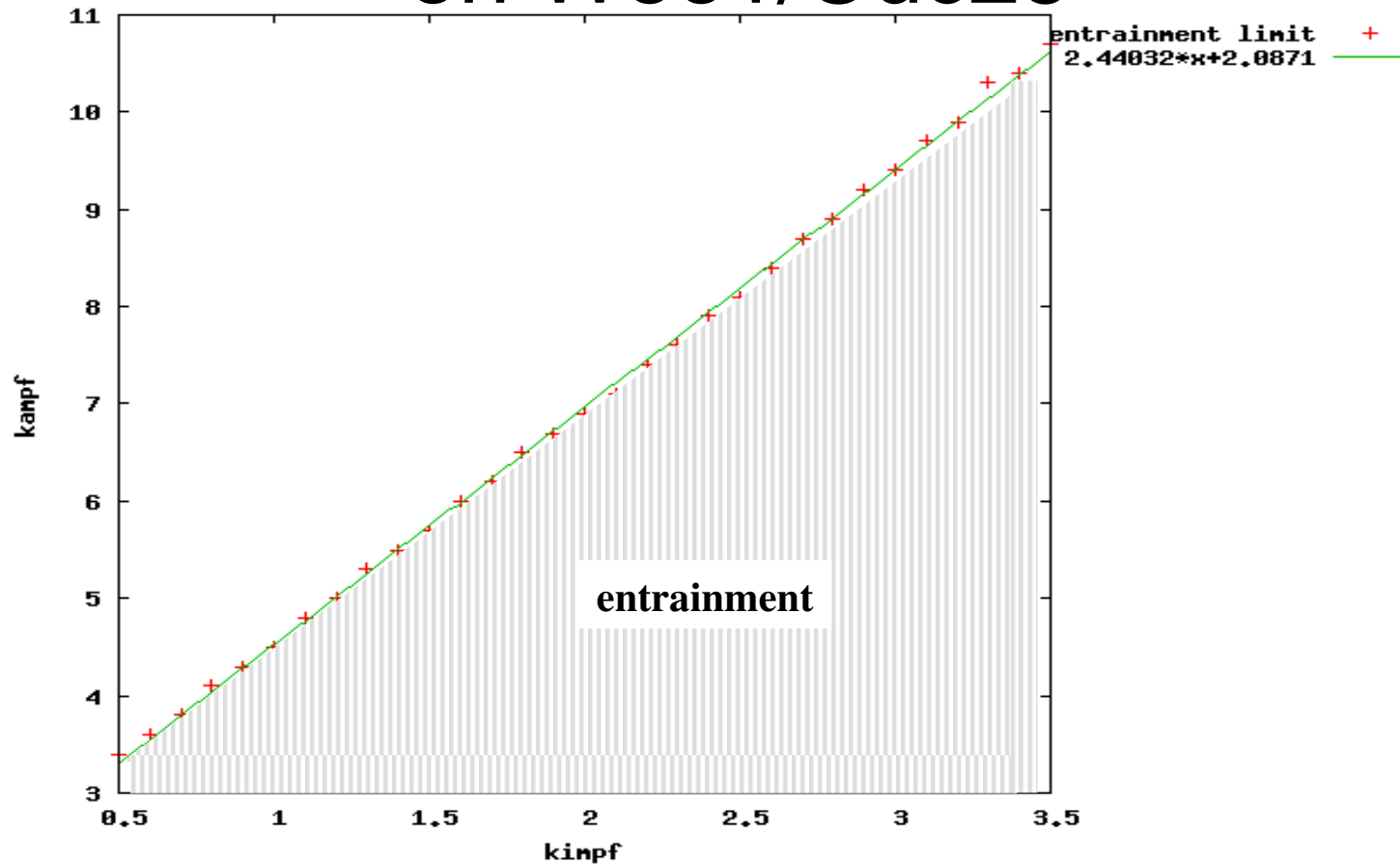


[L. Calzone, S. Soliman 2006]





# Condition of Entrainment in Period on Wee1/Cdc25



Entrainment in period constraint expressed in LTL with the period formula

# Overview of the Tutorial

1. Introduction
  - " Transposing programming concepts to the analysis of living processes
2. Rule-based modeling of biochemical systems
  - " Syntax: molecules, reactions, regulations, SBML/SBGN Biocham notations
  - " Semantics: Boolean, Differential and Stochastic interpretations of reactions
  - " *Static analyses*: consistency, influence graph circuits, protein functions,&
  - " Examples in cell signaling, gene expression, virus infection, cell cycle
3. Temporal Logic based formalization of biological properties
  - " *Qualitative model-checking* in propositional Computation Tree Logic CTL
  - " Quantitative model-checking in Linear Time Logic LTL(R)
  - " *Parameter search* in high dimension w.r.t. LTL(R) specifications
  - " *Robustness and sensitivity analyses* w.r.t. LTL(R) specifications
4. Conclusion