

# Rules2CP and PKML User's Manual

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# Chapter 1

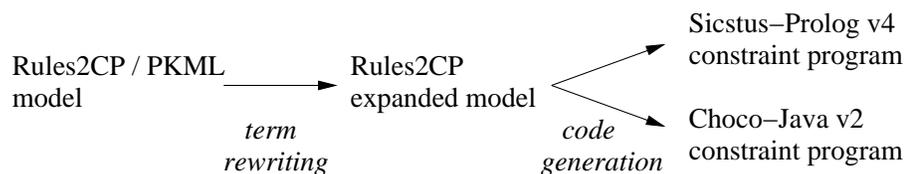
## Getting Started

### 1.1 Overview

Rules2CP is a general purpose rule-based modeling language for constraint programming [4]. It aims at making constraint programming technology easier to use by non-programmers, by modeling combinatorial optimization problems with logical rules and elementary data structures, and by allowing the building of Rules2CP libraries for specific problems.

The Packing Knowledge Modeling Language PKML is such a library developed in the framework of the Net-WMS European project for higher-dimensional bin packing problems taking into account extra placement constraints and specific industrial requirements.

The Rules2CP compiler transforms Rules2CP statements into constraint programs in different target systems, currently Sicstus-Prolog and Choco-Java. More precisely, the Rules2CP compiler implements the following transformations:



This manual describes the syntax and built-in predicates of the Rules2CP modeling language, and the predefined predicates of the PKML library.

Rules2CP is an open-source software distributed under the GPL license and available at <http://contraintes.inria.fr/rules2cp>.

## 1.2 Running Rules2CP

The Rules2CP v1 compiler has two different target languages: Sicstus Prolog 4.0.2 [3] and Choco Java 2.0.0.3 [7]

Rules2CP (and PKML) source file names are suffixed by `rcp`.

- For compiling a Rules2CP/PKML model `file.rcp` to a:
  - SICStus-Prolog program: use command `rcp.sav file.rcp [-rcppath path]` to produce the program `file.pl`
  - Choco-Java program: use command `rcpchoco.sav file.rcp [-rcppath path]` to produce the program `file.java`  
where the option `-rcppath` redefines the prefix path used for importing files. The default path is the `rules2cp` installation directory.
- The commands to execute in the installation directory to build the Rules2CP compiler are for:
  - SICStus-Prolog : `rcp.make` (produces `rcp.sav`)
  - Choco-Java : `rcpchoco.make` (produces `rcpchoco.sav`)

## Chapter 2

# Rules2CP Syntax

Rules2CP manipulates the following lexical entities:

- an *ident* is a word beginning with a lower case letter, or any word between simple quotes.
- a *name* is an identifier that can be prefixed by other identifiers for module and package names.
- a *variable* is a word beginning with either an upper case letter or the underscore character `_`.
- a *string* is a sequence of characters between double quotes.
- a *comment* is a sequence of characters beginning with `%` and ending with the end of line. All comments are ignored.

The syntax of Rules2CP is given by the following grammar.

<i>statement</i>	::= <b>import</b> <i>name</i> .	module import
	<i>head</i> = <i>expr</i> .	declaration
	<i>head</i> --> <i>fol</i> .	rule
	? <i>fol</i> .	goal
<i>head</i>	::= <i>ident</i>	
	<i>ident</i> ( <i>variable</i> ,..., <i>variable</i> )	
<i>fol</i>	::= <i>varbool</i>	boolean
	<i>expr</i> <i>relop</i> <i>expr</i>	comparison
	<i>expr</i> <b>in</b> <i>expr</i>	domain
	<i>name</i>	
	<i>name</i> ( <i>expr</i> ,..., <i>expr</i> )	relation
	<b>not</b> <i>fol</i>	negation
	<i>fol</i> <i>logop</i> <i>fol</i>	logical operator
	<b>forall</b> ( <i>variable</i> , <i>expr</i> , <i>fol</i> )	universal quantifier
	<b>exists</b> ( <i>variable</i> , <i>expr</i> , <i>fol</i> )	existential quantifier
	<b>let</b> ( <i>variable</i> , <i>expr</i> , <i>fol</i> )	variable binding
	<b>foldr</b> ( <i>variable</i> , <i>expr</i> , <i>logop</i> , <i>expr</i> , <i>expr</i> )	logical fold right
	<b>foldl</b> ( <i>variable</i> , <i>expr</i> , <i>logop</i> , <i>expr</i> , <i>expr</i> )	logical fold left
<i>expr</i>	::= <i>varint</i>	
	<i>fol</i>	reification
	<i>string</i>	
	[ <i>enum</i> ]	list
	{ <i>name</i> = <i>expr</i> ,..., <i>name</i> = <i>expr</i> }	record
	<i>name</i>	
	<i>name</i> ( <i>expr</i> ,..., <i>expr</i> )	function
	<i>expr</i> <i>op</i> <i>expr</i>	
	<b>foldr</b> ( <i>variable</i> , <i>expr</i> , <i>op</i> , <i>expr</i> , <i>expr</i> )	fold left
	<b>foldl</b> ( <i>variable</i> , <i>expr</i> , <i>op</i> , <i>expr</i> , <i>expr</i> )	fold right
	<b>map</b> ( <i>variable</i> , <i>expr</i> , <i>expr</i> )	list mapping
<i>enum</i>	::= <i>enum</i> , <i>enum</i>	enumeration
	<i>expr</i>	value
	<i>expr</i> .. <i>expr</i>	interval of integers
<i>varint</i>	::= <i>variable</i>	
	<i>integer</i>   <b>min_integer</b>   <b>max_integer</b>	integers
<i>varbool</i>	::= <i>variable</i>	
	0	false
	1	true
<i>op</i>	::= +   -   *   /   <b>min</b>   <b>max</b>   <b>abs</b>   <b>log</b>   <b>exp</b>	arithmetic
<i>relop</i>	::= <   =<   =   #   >=   >	arithmetic comparisons
<i>logop</i>	::= <b>and</b>   <b>or</b>   <b>implies</b>   <b>equiv</b>   <b>xor</b>	logical connectives
<i>name</i>	::= <i>ident</i>	
	<i>name</i> : <i>ident</i>	module prefix

## Chapter 3

# Data structures with their Built-in Functions and Predicates

The only data structures are booleans, integers, finite domain variables, strings, enumerated lists and records.

### 3.1 Booleans

The Boolean constants `true` and `false` are represented by the integers 1 and 0 respectively.

The usual Boolean operations are described in the syntax table 2 under the item *logop*.

### 3.2 Integers

The integer constants are noted as usual, e.g. -2, 0, 42...

The arithmetic operations and the usual ordering relations on integers are described in the syntax table under the items *op* and *relop* respectively.

### 3.3 Finite Domain Variables

A Rules2CP variable represents either a parameter of a function or predicate, or an unknown integer or boolean, called a finite domain (FD) variable.

A FD variable can be given an initial domain as a list of integers or intervals, with the following built-in predicates:

- *X in list*
  - constrains the variable *X* to take integer values in a list of integer values.

- `domain(expr, min, max)`
  - constrains the list of variables occurring in the expression *expr* to take integer values between *min* and *max*.

The arithmetic operators described above can be used with FD variables to create arithmetic constraints. Furthermore, the following built-in constraints are available:

- `all_different(list)`
  - where the argument is a list of FD variables or integers. The constraint holds if the elements of the list are all different.
- `lexicographic(list)`
  - where the argument is a list of lists of FD variables or integers. The constraint holds if the lists are in ascending or equal lexicographic order.
- `lexicographic_strict(list)`
  - where the argument is a list of lists of FD variables or integers. The constraint holds if the lists are in strictly ascending lexicographic order.
- `non_overlapping(list of PKML objects, List of Dimensions)`
  - constrains a list of PKML objects to non-overlap in a given list of dimensions (see Section 11.3). The FD variables of this constraint are the coordinates of the objects. This built-in constraint uses the global constraint `geost` of the target systems [2].

The following built-in functions return information on the domain of the FD variables. These functions cannot be evaluated statically but can be called under the scope of the `dynamic` predicate.

- `domain_min(X)`
  - returns the lower bound of the domain of *X*.
- `domain_max(X)`
  - returns the upper bound of the domain of *X*.
- `domain_size(X)`
  - returns the size of the domain of variable *X*.

## 3.4 Strings

A string is a sequence of characters between double quotes.

## 3.5 Lists

Lists are formed by enumerating all their elements between brackets. For instance `[1, 3, 4, 5, 6, 8]` is a list of integers which can also be written as a list of intervals as `[1,3..6,8]`. There is no binary list constructor.

The following built-in functions are predefined on lists:

- `length(list)`

returns the length of the list (after expansion of the intervals). It is an error if the argument is not a list.

- `nth(integer,list)`

returns the element of the list in the position (counting from 1) indicated by the first argument, or an error if the second argument is not a list containing the first argument.

- `pos(element,list)`

returns the first position of an element occurring in a list as an integer (counting from 1), or returns an error if the element does not belong to the list.

- `variables(expr)`

returns the list of finite domain variables contained in an expression, i.e. occurring as attributes of a record, or recursively in a record referenced by attributes, in a list, or in a first-order formula.

Furthermore, the following functions on lists of integers are predefined as follows (in library `lib/common/rcp.rcp`):

- `sum(L) = foldr(X, L, +, 0, X)`.

Returns the sum of integers contained in the list L.

- `product(L) = foldr(X, L, *, 1, X)`.

Returns the product of integers contained in the list L.

- `maximum(L) = foldr(X, L, max, inf, X)`.

Returns the greatest integer contained in the list L.

- `minimum(L) = foldr(X, L, min, sup, X)`.

Returns the least integer contained in the list L.

## 3.6 Records

Records are constructed by enumerating their attribute names and values between braces with expressions of the form `{ident = expr,...,ident= expr}`.

For instance `{start=_, duration=2}` is a record representing a task with a variable start point and a fixed duration.

All records have an implicit integer attribute `uid`. This attribute provides a unique identifier for each record.

The attribute value of a record is accessed with the following built-in function:

- `attribute(record)`

returns the expression associated to an attribute name of a record, or returns an error if the argument is not a record or does not have this attribute.

## Chapter 4

# User-defined Functions and Predicates

### 4.1 Functions defined by Declarations

New *functions* can be defined with declarations of the form

- $head = expr.$ 
  - defines the head as a shorthand for the right-hand side expression.

In a declaration, a variable occurring in the body expression and not in the head is a finite domain variable representing an unknown of the problem.

### 4.2 Predicates defined by Rules

New *predicates* can be defined with rules of the form

- $head \rightarrow fol.$ 
  - defines the head as a shorthand for the right-hand side formula.

In a rule, the variables in the body formula are assumed to appear in the head. A rule thus cannot introduce finite domain variables.



## Chapter 5

# Combinators

Rules2CP does not allow recursion in declarations and rules. Built-in combinators are thus available to define various iterations. Combinators cannot be defined in first-order logic and are thus Rule2CP built-ins.

The first-argument of a combinator is a variable  $X$  used to denote place holders in an expression. The second argument is an expression or a list representing the unique or successive values of  $X$  in the expression formed according to the following arguments.

### 5.1 Let

- $\text{let}(X, e, \phi) = \phi[X/e]$

substitutes  $X$  for  $e$  in  $\phi$

### 5.2 Map

- $\text{map}(X, [e_1, \dots, e_N], \phi) = [\phi[X/e_1], \dots, \phi[X/e_N]]$

where  $\phi[X/e]$  denotes the expression  $\phi$  where the free occurrences of  $X$  have been replaced by  $e$ .

### 5.3 Forall, Exists

- $\text{forall}(X, [e_1, \dots, e_N], \phi) = \phi[X/e_1] \wedge \dots \wedge \phi[X/e_N]$
- $\text{exists}(X, [e_1, \dots, e_N], \phi) = \phi[X/e_1] \vee \dots \vee \phi[X/e_N]$

## 5.4 Fold right, Fold left

- $\text{foldr}(X, [e_1, \dots, e_N], op, e, \phi) = \phi[X/e_1] op (\dots op (\phi[X/e_N] op e))$

iteratively combines the first element of the list with the result of the combination of the tail of the list

- $\text{foldl}(X, [e_1, \dots, e_N], op, e, \phi) = ((e op \phi[X/e_1]) op \dots) op \phi[X/e_N]$

iteratively combines the result of the combination of the first elements of the list with the last element of the list

The most general combinators are the left and right fold combinators. The `let`, `forall` and `exists` combinators are defined for convenience but are equivalent to the following folds:

```
let(X, E, F) = foldr(X, [E], and, 1, F)
forall(X, L, E) = foldr(X, L, and, 1, F)
exists(X, L, E) = foldr(X, L, or, 0, F)
```

# Chapter 6

## Search

In Rules2CP, decision variables and branching formulae of the problem are specified in a declarative manner, as well as heuristics as preference orderings.

### 6.1 Enumeration of Variables

- `labeling(expr)`

specifies the enumeration of the possible values of all the variables *contained* in an expression.

### 6.2 And/Or Search Trees

- `search(fol)`

specifies a *branching formula*, i.e. an and/or search tree explored by branching on all the disjunctions, implications and existential quantifications occurring in the formula.

The negations in the formula are eliminated by descending them to the constraints. In order to avoid an exponential growth of the formulae, `equiv` and `xor` formulae are kept as constraints and are not treated as choice points.

### 6.3 Optimization

- `minimize(fol,expr)`, `maximize(fol,expr)`

specifies a *branching formula* (like `search`) together with an optimization criterion given as an expression.



# Chapter 7

## Heuristics

Heuristics for guiding the search are stated in Rules2CP as preference orderings on choice points and branches.

Two pairs of predicates are predefined for specifying choice criteria between variables and values for `labeling`, and between conjunctive and disjunctive formulae for `search`.

### 7.1 Variable and Value Choice Heuristics for Labeling

- `variable_ordering([ident(expr), ..., ident(expr)])`

*ident* is an identifier among the following :

- `greatest` for selecting variables in descending order of *expr* value,
- `least` for selecting variables in ascending order of *expr* value,
- `any` for selecting variables for which the *expr* applies, independently of its value,
- `is` for selecting a variable if it is equal to the *expr* value.

The expression *expr* in a criterion contains the symbol `^` for denoting, for any variable, the left-hand side of the Rules4CP declaration that introduced that variable. If the expression cannot be evaluated on a given variable, the criterion is ignored.

The list of criteria is used for ordering the variables in `labeling` predicates (see Example 10.1). The variables are sorted according to the first criterion when it applies, then the second, etc. The variables for which no criterion applies are considered at the end for labeling in the syntactic order.

- `value_ordering([ident(expr), ..., ident(expr)])`

*ident* is an identifier among the following :

- **up** for enumerating values in ascending order of *expr* value,
- **down** for enumerating values in descending order of *expr* value,
- **step** for binary choices,
- **enum** for multiple choices,
- **bisect** for dichotomy choices,

and where *expr* is an expression containing the symbol  $\wedge$  which denotes the left-hand side of the Rules 2CP declaration that introduces a given variable.

A criterion applies to a variable if it matches the expression *expr*.

## 7.2 Conjunct and Disjunct Choice Heuristics for Search

In search trees defined by logical formulae, the criteria for `conjunct_ordering` and `disjunct_ordering` heuristics are defined similarly by pattern matching on the rule heads that introduce conjunctive and disjunctive formulae under the `search` predicate. This is illustrated in Example 10.2 with conditional expressions of the form `if  $\wedge$  is  $\phi$` ; where  $\wedge$  denotes the conjunct or disjunct candidate for matching  $\phi$ , and  $\phi$  denotes either a rule head or directly a formula. The conjuncts or disjuncts for which no criterion applies are considered last, in the syntactic order.

- `conjunct_ordering([ident(expr), ..., ident(expr)])`

*ident* is identifier among the following :

- **greatest** for selecting conjuncts in descending order of *expr* value,
- **least** for selecting conjuncts in ascending order of *expr* value,

A criterion applies to a conjunct if it matches the expression `name(expr1, ..., exprn)`.

- `disjunct_ordering([ident(expr), ..., ident(expr)])`

*ident* is an identifier among the following :

- **greatest** for selecting disjuncts in descending order of *expr* value,
- **least** for selecting disjuncts in ascending order of *expr* value,

where *expr* is a conditional expression of the form :

`expr if  $\wedge$  is name(expr1, ..., exprn)`

where the symbol  $\wedge$  denotes a Rules2CP predicate of arity *n*.

A criterion applies to a disjunct if it matches the expression `name(expr1, ..., exprn)`.

## Chapter 8

# Interpreter

It happens that combinatorial problems can be decomposed and that the definition of one sub-problem depends on the value or bounds of the FD variables of another component (see example 12.2). As the values of FD variables are unknown at compile-time, the term expansion cannot take place due to a lack of instantiation and would produce an error in the compiler. This is precisely when we need to use the Rules2CP interpreter with the `dynamic` predicate.

- `dynamic(fol)`

prevents the compiler from expanding term *fol*, postpones its evaluation to run time in interpreted mode.

The only Rules2CP expressions that evaluate differently at run time than at compile time are the FD variable domain built-ins. The `dynamic` predicate should thus be used to evaluate the Rules2CP expressions that depend on the bounds (or value) of an FD variable.

The `dynamic` predicate can also be used to limit the size of the generated code.

Note that in version 1.0 of the Rules2CP compiler, the interpreter (i.e. the `dynamic` predicate) is implemented in SICStus Prolog but is not available in Choco Java.



## Chapter 9

# Error Messages

- ... not parsed as ...  
Syntax error, the statement is ill-formed.
- ... is unknown.  
Unknown function or predicate.
- ... should had been reduced to ...  
Type error, the expression is not of the expected type.
- unbound variable ...  
Variable instantiation error.
- ... is not accepted by ... language  
Function or predicate not implemented for the target language.



## Chapter 10

# Simple Rules2CP Examples

### 10.1 NQueens

This is a standard combinatorial puzzle introduced by Bezzel in 1848, for putting  $N$  queens on a chessboard of size  $N \times N$  such that they do not attack each other, i.e. they are not placed on the same row, column or diagonal.

```
q(I) = {row = _, column = I}.
```

```
board(N) = map(I, [1..N], q(I)).
```

```
safe(L) -->
  all_different(L) and
  forall(Q, L,
    forall(R, L,
      let(I, column(Q),
        let(J, column(R),
          I < J implies
            row(Q) # J - I + row(R) and
            row(Q) # I - J + row(R)
          )))
  )).
```

```
queens_constraints(N) -->
  let(B, board(N),
    domain(B, 1, N) and safe(B)).
```

```
queens_search(N) -->
  let(B, board(N),
    variable_ordering([least(domain_size(row(^)))])) and
    labeling(B)).
```

```
? let(N, 4, queens_constraints(N) and queens_search(N) ).
```

## 10.2 Disjunctive Scheduling

Scheduling problems are optimization problems in which we are interested in computing an ordering of the tasks that minimizes the start date of the last task. Each task is given a duration and a time window for its start date. There are precedence constraints between tasks. Furthermore, disjunctive scheduling problems include mutual exclusion constraints (for tasks sharing a same resource) which make the disjunctive scheduling problem NP-hard in general.

```
t1 = {start=_, duration=2}.
t2 = {start=_, duration=5}.
t3 = {start=_, duration=4}.
t4 = {start=_, duration=3}.
t5 = {start=_, duration=1}.

cost = start(t5).

prec(T1, T2) -->
    start(T1) + duration(T1) =< start(T2).

disj(T1, T2) -->
    prec(T1, T2) or prec(T2, T1).

precedences -->
    prec(t1, t2) and prec(t2, t5) and prec(t1, t3) and
    prec(t3, t5) and prec(t1, t3) and prec(t3, t5).

disjunctives -->
    disj(t2, t4) and disj(t3, t4) and disj(t2, t3).

? domain([t1, t2, t3, t4, t5], 0, 20) and
  precedences and
  minimize(disjunctives, cost) and
  conjunct_ordering([greatest(duration(A)+duration(B) if ^ is disj(A,B))]).
```

## 10.3 Bridge Problem

This is the classical disjunctive scheduling problem for the construction of a bridge [8].

```
first = {start=_, duration=0}.

a1 = {start=_, duration=4}.
a2 = {start=_, duration=2}.
a3 = {start=_, duration=2}.
a4 = {start=_, duration=2}.
```

a5 = {start=\_, duration=2}.  
a6 = {start=\_, duration=5}.  
  
p1 = {start=\_, duration=20}.  
p2 = {start=\_, duration=13}.  
  
ue = {start=\_, duration=10}.  
  
s1 = {start=\_, duration=8}.  
s2 = {start=\_, duration=4}.  
s3 = {start=\_, duration=4}.  
s4 = {start=\_, duration=4}.  
s5 = {start=\_, duration=4}.  
s6 = {start=\_, duration=10}.  
  
b1 = {start=\_, duration=1}.  
b2 = {start=\_, duration=1}.  
b3 = {start=\_, duration=1}.  
b4 = {start=\_, duration=1}.  
b5 = {start=\_, duration=1}.  
b6 = {start=\_, duration=1}.  
  
ab1 = {start=\_, duration=1}.  
ab2 = {start=\_, duration=1}.  
ab3 = {start=\_, duration=1}.  
ab4 = {start=\_, duration=1}.  
ab5 = {start=\_, duration=1}.  
ab6 = {start=\_, duration=1}.  
  
m1 = {start=\_, duration=16}.  
m2 = {start=\_, duration=8}.  
m3 = {start=\_, duration=8}.  
m4 = {start=\_, duration=8}.  
m5 = {start=\_, duration=8}.  
m6 = {start=\_, duration=20}.  
  
l1 = {start=\_, duration=2}.  
  
t1 = {start=\_, duration=12}.  
t2 = {start=\_, duration=12}.  
t3 = {start=\_, duration=12}.  
t4 = {start=\_, duration=12}.  
t5 = {start=\_, duration=12}.  
  
ua = {start=\_, duration=10}.  
v1 = {start=\_, duration=15}.

```

v2 = {start=_, duration=10}.
k1 = {start=_, duration=0}.
k2 = {start=_, duration=0}.

last = {start=_, duration=0}.

cost = start(last).

end(Task) = start(Task) + duration(Task).

tasks = [first, a1, a2, a3, a4, a5, a6, p1, p2, ue, s1, s2, s3, s4, s5, s6,
        b1, b2, b3, b4, b5, b6, ab1, ab2, ab3, ab4, ab5, ab6,
        m1, m2, m3, m4, m5, m6,
        l1, t1, t2, t3, t4, t5, ua, v1, v2, k1, k2, last].

precedences_list = [
    [first, a1], [first, a2], [first, a3], [first, a4],
    [first, a5], [first, a6], [first, ue],
    [a1, s1], [a2, s2], [a5, s5], [a6, s6], [a3, p1], [a4, p2],
    [p1, s3], [p2, s4], [p1, k1], [p2, k1],
    [s1, b1], [s2, b2], [s3, b3], [s4, b4], [s5, b5], [s6, b6],
    [b1, ab1], [b2, ab2], [b3, ab3], [b4, ab4], [b5, ab5], [b6, ab6],
    [ab1, m1], [ab2, m2], [ab3, m3], [ab4, m4], [ab5, m5], [ab6, m6],
    [m1, t1], [m2, t1], [m2, t2], [m3, t2], [m3, t3], [m4, t3], [m4, t4], [m5, t4],
    [m5, t5], [m6, t5], [m1, k2], [m2, k2], [m3, k2], [m4, k2], [m5, k2], [m6, k2],
    [l1, t1], [l1, t2], [l1, t3], [l1, t4], [l1, t5],
    [t1, v1], [t5, v2], [t2, last], [t3, last], [t4, last],
    [v1, last], [v2, last], [ua, last], [k1, last], [k2, last]].

resource_crane = [l1, t1, t2, t3, t4, t5].
resource_bricklaying = [m1, m2, m3, m4, m5, m6].
resource_schal = [s1, s2, s3, s4, s5, s6].
resource_excavator = [a1, a2, a3, a4, a5, a6].
resource_ram = [p1, p2].
resource_pump = [b1, b2, b3, b4, b5, b6].
resource_caterpillar = [v1, v2].

resources_list = [resource_crane,
                 resource_bricklaying,
                 resource_schal,
                 resource_excavator,
                 resource_ram,
                 resource_pump,
                 resource_caterpillar].

max_nf_list = [[first, l1, 30], [a1, s1, 3], [a2, s2, 3],

```

```

        [a5, s5, 3], [a6, s6, 3], [p1, s3, 3], [p2, s4, 3]].
min_sf_list = [[ua, m1, 2], [ua, m2, 2], [ua, m3, 2], [ua, m4, 2],
               [ua, m5, 2], [ua, m6, 2]].
max_ef_list = [[s1, b1, 4], [s2, b2, 4], [s3, b3, 4], [s4, b4, 4],
               [s5, b5, 4], [s6, b6, 4]].
min_nf_list = [[first, l1, 30]].
min_af_list = [[ue, s1, 6], [ue, s2, 6], [ue, s3, 6], [ue, s4, 6],
               [ue, s5, 6], [ue, s6, 6]].

%% rules

prec(T1, T2) -->
    end(T1) =< start(T2).

disj(T1, T2) -->
    prec(T1, T2) or prec(T2, T1).

precedences -->
    forall(TaskPair, precedences_list,
           prec(nth(1, TaskPair), nth(2, TaskPair))).

disjunctives -->
    forall(Tasks, resources_list,
           forall(T1, Tasks,
                  forall(T2, Tasks,
                         uid(T1) < uid(T2) implies disj(T1, T2)))).

max_nf(T1,T2, N) -->
    start(T2) =< end(T1) + N.

min_nf(T1, T2, N) -->
    start(T2) >= start(T1) + 2 * duration(T1) + N.

max_ef(T1, T2, N) -->
    end(T2) =< end(T1) + N.

min_af(T1, T2, N) -->
    start(T2) >= start(T1) + N.

min_sf(T1, T2, N) -->
    end(T2) =< start(T1) + N.

distances -->
    forall(T, max_nf_list, max_nf(nth(1, T), nth(2, T), nth(3, T))) and
    forall(T, min_sf_list, min_sf(nth(1, T), nth(2, T), nth(3, T))) and
    forall(T, max_ef_list, max_ef(nth(1, T), nth(2, T), nth(3, T))) and

```

```
forall(T, min_nf_list, min_nf(nth(1, T), nth(2, T), nth(3, T))) and
forall(T, min_af_list, min_af(nth(1, T), nth(2, T), nth(3, T))).
```

tasks\_domain -->

```
domain(map(T, tasks, start(T)), 0, 200).
```

? tasks\_domain and precedences and distances and

```
minimize(disjunctives, cost) and
```

```
conjunct_ordering([greatest(duration(A) + duration(B) if ^ is disj(A, B))]) and
```

```
disjunct_ordering([greatest(duration(A) if ^ is prec(A, B))]).
```

## Chapter 11

# The Packing Knowledge Modelling Language PKML

The Packing Knowledge Modelling Language (PKML) developed in the EU FP6 Net-WMS project is defined as a Rules2CP library. This makes PKML easily extensible with new features and customizable for particular needs.

The PKML package includes a library for dealing with Allen's interval relations in one dimension [1], another library for dealing with region connection calculus relations in an arbitrary number of dimensions [5], and a proper PKML library which defines higher-dimensional objects with alternative shapes, each shape being a rigid assembly of boxes.

Rules and strategies for solving pure bin packing problems as well as packing problems with specific requirements on the shape and weights of items to pack, are predefined in the PKML library.

### 11.1 Allen's Interval Relations in One Dimension

In one dimension, the library of Allen's interval relations between objects is predefined in Rules2CP in the following file `allen.rcp`:

```
% Copyright 2008 INRIA, F. Fages & J. Martin
% License GPL
%
% file allen.rcp
%

precedes(A, B, D) -->
    end(A, D) < origin(B, D).

meets(A, B, D) -->
    end(A, D) = origin(B, D).
```

```

overlaps(A, B, D) -->
    origin(A, D) < origin(B, D) and
    end(A, D) < end(B, D) and
    origin(B, D) < end(A, D).

contains(A, B, D) -->
    origin(A, D) < origin(B, D) and
    end(B, D) < end(A, D).

starts(A, B, D) -->
    origin(A, D) = origin(B, D) and
    end(A, D) < end(B, D).

finishes(A, B, D) -->
    origin(B, D) < origin(A, D) and
    end(A, D) = end(B, D).

equals(A, B, D) -->
    origin(A, D) = origin(B, D) and
    end(A, D) = end(B, D).

started_by(A, B, D) -->
    origin(A, D) = origin(B, D) and
    end(B, D) < end(A, D).

finished_by(A, B, D) -->
    origin(B, D) > origin(A, D) and
    end(A, D) = end(B, D).

during(A, B, D) -->
    origin(B, D) < origin(A, D) and
    end(A, D) < end(B, D).

overlapped_by(A, B, D) -->
    origin(B, D) < origin(A, D) and
    origin(A, D) < end(B, D) and
    end(A, D) > end(B, D).

met_by(A, B, D) -->
    end(B, D) = origin(A, D).

preceded_by(A, B, D) -->
    end(B, D) < origin(A, D).

contains_touch(A, B, D) -->

```

```

origin(A, D) =< origin(B, D) and
end(B, D) =< end(A, D).

```

```

overlaps_sym(A, B, D) -->
    end(A, D) > origin(B, D) and
    end(B, D) > origin(A, D).

```

The predicate `contains_touch` and `overlaps_sym` have been added to Allen's relations. These relations can be defined by disjunctions of standard Allen's relations but their direct definition by conjunctions of inequalities is added here for efficiency reasons.

## 11.2 Region Connection Calculus in Higher-Dimensions

In higher-dimensions, the library of topological relations of the Region Connection Calculus [5] is predefined in Rules2CP between objects. For the sake of simplicity of the following file `rcc8.rcp`, the assemblies of boxes are treated as the least box containing the assembly, using the `size(S,D)` function.

```

% Copyright 2008 INRIA, F. Fages & J. Martin
% License GPL
%
% file rcc.rcp
%

import('rules2cp/lib/pkml/allen').

disjoint(O1, O2, Ds) -->
    exists(D, Ds,
        precedes(O1, O2, D) or
        preceded_by(O1, O2, D)).

meet(O1, O2, Ds) -->
    forall(D, Ds,
        not precedes(O1, O2, D) and
        not preceded_by(O1, O2, D)) and
    exists(D, Ds,
        meets(O1, O2, D) or
        met_by(O1, O2, D)).

equal(O1, O2, Ds) -->
    forall(D, Ds, equals(O1, O2, D)).

covers(O1, O2, Ds) -->
    forall(D, Ds,
        started_by(O1, O2, D) or
        contains(O1, O2, D) or

```

```

        finished_by(01, 02, D)) and
exists(D, Ds, not contains(01, 02, D)).

covered_by(01, 02, Ds) -->
    forall(D, Ds,
        starts(01, 02, D) or
        during(01, 02, D) or
        finishes(01, 02, D)) and
exists(D, Ds, not during(01, 02, D)).

contains_rcc(01, 02, Ds) -->
    forall(D, Ds, contains(01, 02, D)).

inside(01, 02, Ds) -->
    forall(D, Ds, during(01, 02, D)).

overlap(01, 02, Ds) -->
    forall(D, Ds, overlaps_sym(01, 02, D)).

contains_touch_rcc(01, 02, Ds) -->
    forall(D, Ds, contains_touch(01, 02, D)).

```

The rule `contains_touch_rcc` has been added to the standard region calculus connection relations for convenience and efficiency reasons similar to the extension done to Allen's relations.

### 11.3 PKML Library

The PKML library is defined in Rules2CP by the following file `lib/pkml/pkml.rcp`:

```

% Copyright 2008 INRIA, F. Fages & J. Martin
% License GPL
%
% file pkml.rcp
%

import('rules2cp/lib/common/rcp').
import('rules2cp/lib/pkml/rcc').
import('rules2cp/lib/pkml/pkml_surface').
import('rules2cp/lib/pkml/pkml_weight').
%import('rules2cp/lib/pkml/pkml_gui').

%% Boxes
%
% boxes given with their size in each dimension
% b = {size = [s1, ..., sk]}

```

```

make_box(L) = {size = L}.

box_volume(B) = product(size(B)).

%% Shifted Boxes
%
% shifted boxes given with their box and their offset
% b = {box = b, offset = [o1,..,ok]}

make_sbox(B, 0) = {box = B, offset = 0}.

sbox_size(SB, D) = nth(D, size(box(SB))).

sbox_offset(SB, D) = nth(D, offset(SB)).

sbox_end(SB, D) = sbox_offset(SB, D) + sbox_size(SB, D).

%% Shapes
%
% shapes as assemblies of boxes given with their positions
% shape = {sboxes=[sb1,..,sbm]}
%
% - make_shape_box constructor for a single box
% - size of a shape in one dimension as the maximum size of its assembly
% - shape_volume is the volume of an assembly shape
% (overapproximation if sboxes overlap)

make_shape(SBs) = {sboxes = SBs}.

make_shape_box(L) = make_shape([make_sbox(make_box(L), map(_, L, 0))]).

shape_volume(S) = sum(map(SB, sboxes(S), box_volume(box(SB)))).

shape_origin(S, D) = minimum(map(SB, sboxes(S), sbox_offset(SB, D))).

shape_end(S, D) = maximum(map(SB, sboxes(S), sbox_end(SB, D))).

shape_size(S, D) = shape_end(S, D) - shape_origin(S, D).

%% Objects
%
% objects with alternative shapes
% object = {shapes=[s1,..,sN], shape_index=_, origin=[X1,..,Xk]}
%
% - object constructors with a single shape

```

```

% - shape domain of an object
% - origin of an object
% - x,y,z coordinates of an object
% - end of an object with alternative shapes
% - volume of an object as the volume of its shape

make_object(SL, OL) = {shapes=SL, shape_index=S, origin=OL}.

make_object(SL, OL, S, W) = {shapes=SL, shape_index=S, origin=OL, weight=W}.

make_object_shape(S, L) = {shapes=[S], shape_index=1, origin=L}.

make_object_shape(S, L, W) = {shapes=[S], shape_index=1, origin=L, weight=W}.

object_shape_domain(O) --> shape_index(O) in [1 .. length(shapes(O))].

object_shape_domains(Items) --> forall(I, Items, object_shape_domain(I)).

origin(O, D) = nth(D, origin(O)).

x(O) = origin(O, 1).

y(O) = origin(O, 2).

z(O) = origin(O, 3).

end(O, D) = origin(O, D) +
            sum(map(S, shapes(O),
                    (shape_index(O) = pos(S, shapes(O))) * shape_end(S, D))).

size(O, D) = sum(map(S, shapes(O),
                    (shape_index(O) = pos(S, shapes(O))) * shape_size(S, D))).

volume(O) = sum(map(S, shapes(O),
                    (shape_index(O) = pos(S, shapes(O))) * shape_volume(S))).

distance(O1, O2, D) = max(0, max(origin(O1, D), origin(O2, D))
                        - min(end(O1, D), end(O2, D))).

%%
% Rules for pure bin packing problems
non_overlapping_binary(Items, Dims) -->
    forall(O1, Items,
            forall(O2, Items,
                    uid(O1) < uid(O2) implies
                    not overlap(O1, O2, Dims))).

```

```

containmentAE(Items, Bins, Dims) -->
    forall(I, Items,
        exists(B, Bins,
            contains_touch_rcc(B, I, Dims))).

bin_packing_binary(Items, Bins, Dims) -->
    containmentAE(Items, Bins, Dims) and
    non_overlapping_binary(Items, Dims) and
    labeling(Items).

bin_packing(Items, Bins, Dims) -->
    containmentAE(Items, Bins, Dims) and
    non_overlapping(Items, Dims) and
    labeling(Items).

%%
% Rules for pure bin design problems
containmentEA(Items, Bins, Dims) -->
    exists(B, Bins, forall(I, Items, contains_touch_rcc(B,I,Dims))).

bin_design(Bin, Items, Dims) -->
    containmentEA(Items, [Bin], Dims) and
    non_overlapping(Items, Dims) and
    minimize(labeling(Items), volume(Bin)).

```

These rules allow us to express pure bin packing and pure bin design problems.

The file `pkml_weight.rcp` defines some additional common sense rules of packing taking into account the weight of items:

```

% Copyright 2008 INRIA, F. Fages & J. Martin
% License GPL
%
% file pkml_weight.rcp
%

lighter(O1, O2) -->
    weight(O1) =< weight(O2).

heavier(O1, O2) -->
    weight(O1) >= weight(O2).

gravity(Items) -->
    forall(O1, Items,
        origin(O1, 3) = 0 or
        exists(O2, Items, uid(O1) # uid(O2) and on_top(O1, O2))).

```

```

weight_stacking(Items) -->
  forall(O1, Items,
    forall(O2, Items,
      (uid(O1) # uid(O2) and above(O1, O2))
      implies
      lighter(O1, O2))).

weight_balancing(Items, Bin, D, Ratio) -->
  let(L, sum( map(I1, Items, weight(I1) * (end(I1,D) =< (end(Bin, D) / 2)))),
    let(R, sum( map(Ir, Items, weight(Ir) * (origin(Ir,D) >= (end(Bin, D) / 2)))),
      100*max(L, R) =< (100 + Ratio) * min(L, R))).

```

The file `pkml_surface.rcp` defines some additional rules for taking into account the surface of contact between stacked items:

```

% Copyright 2008 INRIA, F. Fages & J. Martin
% License GPL
%
% file pkml_surface.rcp
%

above(O1, O2) -->
  overlap(O1, O2, [1, 2]) and
  preceded_by(O1, O2, 3) or met_by(O1, O2, 3).

on_top(O1, O2) -->
  overlap(O1, O2, [1, 2]) and
  met_by(O1, O2, 3).

oversize(O1, O2, D) =
  max( max( origin(O1, D), origin(O2, D))
    - min( origin(O1, D), origin(O2, D)),
    max( end(O1, D), end(O2, D))
    - min( end(O1, D), end(O2, D))).

stack_oversize(Items, Length) -->
  forall(O1, Items,
    forall(O2, Items,
      (overlap(O1, O2, [1,2]) and uid(O1) # uid(O2))
      implies
      forall(D, [1,2], oversize(O1, O2, D) =< Length))).

```

## Chapter 12

# PKML Examples

### 12.1 Bin Packing

A small PKML example involving packing business rules taking into account the weight of objects and coming from the automotive industry at Peugeot Citroën PSA, is defined in the following file `psa.rcp`:

```
% Copyright 2008 INRIA, J. Martin & F. Fages
% License GPL
%
% file psa.rcp
%

import('rules2cp/lib/pkml/pkml').

s1 = make_shape_box([1203, 235, 239]).
s2 = make_shape_box([224, 224, 222]).
s3 = make_shape_box([224, 224, 148]).
s4 = make_shape_box([224, 224, 111]).
s5 = make_shape_box([224, 224, 74]).
s6 = make_shape_box([155, 224, 222]).
s7 = make_shape_box([112, 224, 148]).

o1 = make_object_shape(s1, [0, 0, 0]).
o2 = make_object_shape(s4, [_, _, _], 413).
o3 = make_object_shape(s5, [_, _, _], 463).
o4 = make_object_shape(s5, [_, _, _], 842).
o5 = make_object_shape(s3, [_, _, _], 422).
o6 = make_object_shape(s4, [_, _, _], 266).
o7 = make_object_shape(s4, [_, _, _], 321).
o8 = make_object_shape(s2, [_, _, _], 670).
o9 = make_object_shape(s6, [_, _, _], 440).
```

```

o10 = make_object_shape(s7, [_, _, _], 325).

s11 = make_shape_box([_, _, _]).
s41 = make_shape_box([224, 111, 224]).
s51 = make_shape_box([224, 74, 224]).

o11 = make_object_shape(s11, [0,0,0]).
o41 = {shapes=[s4, s41], shape_index=_, origin=[_, _, _], weight=413}.
o51 = {shapes=[s5, s51], shape_index=_, origin=[_, _, _], weight=463}.

bin = o1.
items = [o2, o3, o4, o5, o6, o7, o8, o9, o10].
dimensions = [1, 2, 3].

w(0) = size(0, 1).
h(0) = size(0, 2).
l(0) = size(0, 3).

items_domain(Items, Bin) -->
    forall(I, Items,
        domain(x(I), 0, w(Bin) - w(I)) and
        domain(y(I), 0, h(Bin) - h(I)) and
        domain(z(I), 0, l(Bin) - l(I))).

psa_bin_packing(Bin, Items, Dims) -->
    items_domain(Items, Bin) and
    gravity(Items) and
    weight_stacking(Items) and
    weight_balancing(Items, Bin, 1, 20) and
    stack_oversize(Items, 10) and

    variable_ordering([greatest(weight(^)),
                        greatest(volume(^)),
                        is(z(^))]) and

    value_ordering([up(z(^)),
                    bisect(x(^)),
                    bisect(y(^))]) and

    bin_packing(Items, [Bin], Dims).

% psa_bin_design(Bin, Items, Dims) -->
%     gravity(Items) and
%     bin_design(Bin, Items, Dims) and
%     variable_ordering([greatest(volume(^, Dims)), is(z(^))]).

```

```
? psa_bin_packing(bin, items, dimensions).

%? shape_domains([o41, o51]) and
% psa_bin_design(bin, items, dimensions).
```

## 12.2 Optimal Rectangle Packing

This example is a transcription in PKML of the constraint program used by Simonis and O’Sullivan to solve optimal rectangle packing problems [6]. The search strategy is modeled using the `dynamic` predicate.

In this model, the `disjoint2` and `cumulative` predicates are undocumented constraints used for the sake of comparison with the original program in SICStus Prolog. These constraints are subsumed in Rules2CP by the `geost` predicate.

```
% Rectangle Packing model (based on [Simonis & O’Sullivan, CPAIOR’08])
%
% Find the smallest rectangle (bin) containing
% N squares (items) of sizes 1*1, 2*2, ..., N*N.
%

import('rules2cp/lib/pkml/pkml').

% Data structures & handy macro defs
%
make_object_shape_area(S, OL, A) = {shapes=[S], shape_index=1, origin=OL, area=A}.

bin = make_object_shape_area(make_shape_box([_, _]), [1, 1], _).

{shapes = [], shape_index = 1, origin = [1, 1], area = _}.

items_area(Items) = sum(map(I, Items, volume(I))).

items(N) = map(S, squares(N), item(S)).

item(S) = {shapes = [S], shape_index = 1, origin = [_,_]}.

squares(N) = map(Size, reverse_list([2 .. N]), square(Size)).

square(S) = make_shape_box([S, S]).

w(O) = size(O, 1).
h(O) = size(O, 2).
l(O) = size(O, 3).

xs(Items) = map(I, Items, x(I)).
```

```

ys(Items) = map(I, Items, y(I)).

reverse_list(L) = foldl(X, L, flip(cons), [], X).

% Items subproblem definition
%
solve_items_subproblem(Items, W, H) -->
    state_items_domain(Items, W, H) and
    state_items_constraints(Items, W, H) and
    state_items_search(Items, W, H).

state_items_domain(Items, W, H) -->
    forall(It, Items,
        let(X, x(It),
            let(Y, y(It),
                let(S, w(It),
                    domain(X, 1, W - S + 1) and
                    domain(Y, 1, H - S + 1) )))) and
        lower_quadrant(Items, W, H).

lower_quadrant(Items, W, H) -->
    let(FstIt, nth(1, Items),
        let(X, x(FstIt),
            let(Y, y(FstIt),
                let(S, w(FstIt),
                    domain(X, 1, (W - S + 2) / 2) and
                    domain(Y, 1, (H + 1) / 2) )))).

state_items_constraints(Items, W, H) -->
    let(Xs, map(It, Items, x(It)),
        let(Ys, map(It, Items, y(It)),
            let(Ss, map(It, Items, w(It)),
                disjoint2(Xs, Ys, Ss) and
                cumulative(Xs, Ss, Ss, H) and
                cumulative(Ys, Ss, Ss, W) ))).

state_items_search(Items, W, H) -->
    let(XSs, map(It, Items, {coord = x(It), siz = w(It)}),
        let(YSs, map(It, Items, {coord = y(It), siz = h(It)}),
            let(Min, 1,
                let(MaxX, W + 1,
                    let(MaxY, H + 1,
                        dynamic(
                            search(
                                forall(XS, XSs,
                                    siz(XS) > 6 implies

```

```

        interval_split(coord(XS), Min, MaxX, max(1, (siz(XS) * 3) / 10))
    and
    forall(XS, XSs, dichotomic_split(coord(XS))) and

    forall(YS, YSs,
        interval_split(coord(YS), Min, MaxY, max(1, (siz(YS) * 3) / 10))
    and
    forall(YS, YSs, dichotomic_split(coord(YS)))
    )))))).

% Bin subproblem definition
%
solve_bin_subproblem(W, H, A, L, U, N) -->
    state_bin_domain(W, H, A, L, U, N) and
    state_bin_constraints(W, H, A, N) and
    state_bin_search.

state_bin_domain(W, H, A, L, U, N) -->
    domain([W, H], N, L) and
    A in [L..U].

state_bin_constraints(W, H, A, N) -->
    let(K, (W + 1)/2,
        A = W * H and
        W <= H and
        (W >= 2 * N - 1 or H >= (N * N + N - (K - 1) * (K - 1) - (K - 1)) / 2)).

state_bin_search -->
    labeling(bin) and
    variable_ordering([is(area(^)), is(w(^)), is(h(^))]).

% Search strategy definition
%

% interval splitting
%

interval_split(X, Min, Max, L) -->
    foldl(Cut, map(K, [1..((Max - Min) / (L + 1)) + 1], (Min + K * L + K - 1)),
        interval_predicate, true,
        [X, L]).

interval_predicate(E, List) -->
    let(X, nth(1, List),
        let(L, nth(2, List),
            let(XDLBL, domain_min(X) + L,

```

```

(X =< XDLBL or (X > XDLBL and E))))).

% dychotomic splitting
%

dichotomic_split(X) -->
  let(L, domain_max(X) - domain_min(X) + 1,
      foldl(Cut, [1..log(2, L)], dichotomy_predicate, true, X)).

dichotomy_predicate(List, X) -->
  let(XDM, (domain_min(X) + domain_max(X)) / 2, (X =< XDM) or (X > XDM)).

% Query
%
% N : problem instance size (N = card(Items))
% Items : item records {box(K, K) | forall K in 1..N}
% Width : bin width (length in the first dimension (x))
% Height : bin height (length in the second dimension (y))
% Area : bin area
% LB : optimal solution lower bound
% UB : optimal solution upper bound

? let(N, 17,
     let(Items, items(N),
         let(Width, size(bin, 1),
             let(Height, size(bin, 2),
                 let(Area, area(bin),
                     let(LB, items_area(Items) + 1,
                         let(UB, LB + 200,

solve_bin_subproblem(Width, Height, Area, LB, UB, N) and
solve_items_subproblem(Items, Width, Height) )))))).

```

## 12.3 Bin Packing with Polymorphic Shapes

This small example illustrates the use of polymorphic shapes to represent the possible rotations of an object. In this example the object `o2` (as well as `o3`) can be rotated by 90 degrees in the x-y plane. This is represented by using alternative shapes `s2` and `s3` for this object.

```

% Copyright 2008 INRIA, J. Martin & F. Fages
% License GPL
%
% file psa.rcp

```

```

%

import('rules2cp/lib/pkml/pkml').

s1 = make_shape_box([5, 4, 4]).
s2 = make_shape_box([4, 5, 2]).
s3 = make_shape_box([5, 4, 2]).

o1 = make_object_shape(s1, [0, 0, 0]).
o2 = make_object([s2, s3], [_, _, _], _, 10).
o3 = make_object([s2, s3], [_, _, _], _, 11).

dimensions = [1, 2, 3].
bin = o1.
items = [o2, o3].

w(0) = size(0, 1).
h(0) = size(0, 2).
l(0) = size(0, 3).

items_domain(Items, Bin) -->
    forall(I, Items,
        domain(x(I), 0, w(Bin)) and
        domain(y(I), 0, h(Bin)) and
        domain(z(I), 0, l(Bin))).

? items_domain(items, bin) and
  object_shape_domains(items) and
  bin_packing(items, [bin], dimensions) and
  variable_ordering([greatest(weight(^)), is(shape_index(^)), is(z(^))]).

```



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